PAPER - II : MODEL PAPER - 06

(SPECIMEN PAPER)

MATHEMATICS & STATISTICS

COMMERCE

TIME : 1 HR 30 MIN **MARKS : 40**

Q4. Attempt any six of the following **(12)**

01. Mr Mathew insures his property of 1,00,000 with three insurance companies X , Y and Z for 60,000 , 40,000 and 20,000 respectively . Fire breaks out and causes a loss of 80,000 Calculate the amount that can be claimed from X

SOLUTION

Property value = 1,00,000

Total insured value = 1,20,000 > Property value

$$\text{Claim from Co. X} = \frac{\text{insured value}}{\text{total insured value}} \times \text{loss}$$

$$= \frac{60,000}{1,20,000} \times 80,000$$

$$= 40,000$$

02. if the total population under study is 45,000 and age SDR for the age group (60 and above) is 25 , find CDR and the values of x and y

Age Group	Population	No. of deaths
0 – 30	4000	100
30 – 60	x	150
60&above	y	650

SOLUTION

STEP 1

SDR (30 – 60) = 25

$\frac{D}{P} \times 1000 = 25$

$\frac{150}{x} \times 1000 = 25$

$x = \frac{150 \times 1000}{25}$

$= 6000$

STEP 2

$\Sigma P = 45000$

$4000 + x + y = 45000$

$x + y = 41000$

$6000 + y = 45000$

$y = 39000$

STEP 3

$CDR = \frac{\Sigma D}{\Sigma P} \times 1000$

$= \frac{900}{45000} \times 1000$

= 20 per thousand

03. on an average A can solve 40% of the problems . What is the probability of A solving 4 problems out of 6

SOLUTION

A is given 6 problems ; n = 6

For a trial Success – A solves the problem

p – probability of success = 40/100 = 2/5

q – probability of failure = 1 – 2/5 = 3/5

r.v. x – no of successes = 0 , 1 , 2 , 3 , ... , 6 ; **X ~ B (6 , 2/5)**

P(A solves 4 problems out of 6)

= P(X = 4)

$$= {}^{6}C_2 \left(\frac{2}{5}\right)^4 \left(\frac{3}{5}\right)^2$$

$$= \frac{15.16\ .\ 9}{5^6}$$

$$= \frac{3.16.9}{5^5} \qquad = \frac{432}{3125}$$

04. Amar , akbar and Anthony started a transport business by investing ₹1,00,000 each . Amar left after 5 months from the commencement of the business and Akbar left 3 months later . At the end of the year , the business realized a profit of ₹ 37,500 . Find the Akbar's share of profit

SOLUTION

Profit will be share in the ratio of PERIOD OF INVESTMENT

PSR : Amar : Akbar : Anthony

5 : 8 : 12

Profit = ₹ 37,500

Akbar's share in profit = $\frac{8}{25}$ x 37,500 = ₹ 12,000

05. the income of the broker remained unchanged though the rate of commission increased from 6% to 7.5%. Find the percentage reduction in the value of the business

SOLUTION

let initial sales = 100

commission @ 6% = 6

let new sales = x

commission @ 7.5% = $\frac{7.5\ x}{100}$

Since income of the broker remains unchanged , $\frac{7.5x}{100} = 6$

$$x = \frac{6 \times 1000}{75} = 80$$

% reduction in the value of business = 20%

06. Divija wants to invest at most ₹ 15,000 in Savings Certificates and Fixed Deposits . She wants to invest at least ₹ 3,000 in Savings Certificates and at least ₹ 5,000 in Fixed Deposits . The rate of interest on Saving Certificate is 8% p.a. and that on Fixed Deposit is 10% p.a. FORMULATE the above problem as LPP to determine maximum yearly income .

SOLUTION

Let amount invested in Savings Certificate by x & in Fixed Deposit be y .

CONSTRAINTS

✓ Since Divija wants to invest at most 15,000 in Savings Certificates and Fixed Deposits

$$x + y \le 15000$$

✓ Since Divija wants to invest at least ₹ 3,000 in Savings Certificates and at least ₹ 5,000 in Fixed Deposits

$$x \ge 3000 \quad \& \quad y \ge 5000$$

✓ Since x & y are amounts invested , $x , y \ge 0$

OBJECTIVE FUNCTION

The rate of interest on Saving Certificate is 8% p.a. and that on Fixed Deposit is 10% p.a.

Yearly Interest income = $0.08x + 0.10y$ (in Rs)

Maximize $z = 0.08x + 0.10y$

LPP MODEL

Maximize $z = 0.08x + 0.10y$

Subject to : $x + y \le 15000$; $x \ge 3000$; $y \ge 5000$; $x , y \ge 0$

07. the pdf of continuous random variable X is given by

$$f(x) = \frac{x^2}{18} \quad , \quad -3 < x < 3$$

SOLUTION

$$P(|X| < 1) = P(-1 < x < 1)$$

$$= \int_{-1}^{+1} \frac{x^2}{18} dx$$

$$= \left(\frac{x^3}{54} \right)_{-1}^{+1}$$

$$= \left(\frac{1}{54} \right) - \left(\frac{-1}{54} \right)$$

$$= \frac{2}{54}$$

$$= \frac{1}{27}$$

08. The average number of misprints per page of a book is 1.5 . Assuming the distribution of the number of misprints to be Poisson , find the probability that a particular page of a book is free from misprints ($e^{-1.5} = 0.2231$)

SOLUTION

m = average no of misprints per page

= 1.5

X = number of misprints

$X \sim P(m = 1.5)$

P(particular page is free of misprints)

= P(0)

Using $P(x) = \dfrac{e^{-m}m^x}{x!}$

$$= \frac{e^{-1.5}1.5^0}{0!}$$

$$= e^{-1.5}$$

$$= 0.2231$$

Q5. (A) Attempt any TWO of the following **(06)**

01. Obtain the expected value and variance of a random variable X for the following probability distribution

x	−2	−1	0	1	2	3
P(X = x)	0.1	k	0.2	2k	0.3	k

SOLUTION

STEP 1 $\Sigma p(x) = 1$

$4k + 0.6 = 1$

$4k = 0.4$

$k = 0.1$

STEP 2

x	p(x)	pixi	pixi^2
−2	0.1	−0.2	0.4
−1	0.1	−0.1	0.1
0	0.2	0	0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.1	0.3	0.9
		0.8	2.8

STEP 3

$E(x) = \Sigma pi.xi = 0.8$

$Var(x) = \Sigma pixi^2 - E(x)^2 = 2.8 - (0.8)^2 = 2.8 - 0.64 = 2.16$

03. Calculate the Spearman's rank Correlation coefficient between the following marks given by two judges to 8 contestants in the election elocution

Marks by A	:	81	72	60	33	29	11	56	42
Marks by B	:	75	56	42	15	30	20	60	80

SOLUTION

A	B	x	y	d = x − y	d^2
81	75	1	2	−1	1
72	56	2	4	−2	4
60	42	3	5	−2	4
33	15	6	8	−2	4
29	30	7	6	1	1
11	20	8	7	1	1
56	60	4	3	1	1
42	80	5	1	4	16
					32

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(32)}{8(64 - 1)}$$

$$= 1 - \frac{8}{21}$$

$$= \frac{13}{21}$$

$$= 0.62$$

02. Construct bivariate frequency table for income(X) and expenditure (Y)of 25 families given below taking intervals 200 − 300 ; 300 − 400 ; …. for X & Y

(250,200) ; (300,280) ; (325,300) ; (400,300) ; (450,280)
(325,310) ; (450,325) ; (275,200) ; (355,245) ; (425,375)
(475,400) ; (410,300) ; (280,225) ; (300,250) ; (425,400)
(365,300) ; (270,200) ; (310,210) ; (375,200) ; (345,310)
(290,210) ; (270,215) ; (300,210) ; (425,375) ; (470,380)

Find a) marginal frequency distributions for X and Y

b) conditional freq. dist. of X when Y is between 200 − 300

c) conditional freq. dist. of Y when X is between 400 − 500

SOLUTION

BIVARIATE FREQUENCY DISTRIBUTION TABLE

EXPENDITURE	INCOME –X			
Y	200 − 300	300 − 400	400 − 500	TOTAL
200 – 300	~~||||~~ | 6	~~||||~~ | 6	| 1	13
300 – 400		|||| 4	~~||||~~ | 6	10
400 – 500			|| 2	2
TOTAL	6	10	9	25

CONDITIONAL FREQUENCY DISTRIBUTION OF X WHEN Y IS IN 200 − 300

CI	200–300	300–400	400–500	TOTAL
F	6	6	1	13

CONDITIONAL FREQUENCY DISTRIBUTION OF Y WHEN X IS IN 400 − 500

CI	200–300	300–400	400–500	TOTAL
F	1	6	2	9

(B)

01

A bill of a certain amount drawn on 28th February 2007 for 8 months was cashed on 26th March 2007 for 5,496 at 14% p.a . Find the face value of the bill

SOLUTION due 8 months @ 14 % p.a.

28th Feb	26th Mar	31st Oct
	₹ 5,496	₹ x

STEP 1 :

Date of drawing	=	28 / 02
Add period of bill	+	8 months
Nominal due date	=	28 / 10
Add Grace days	+	3 days
Legal due date	=	31 / 10......

STEP 2 :

Unexpired period

= 26th Mar – 31th Aug

MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT

= 5 + 30 + 31 + 30 + 31 + 31 + 30 + 31

= 219 days

STEP 3 :

B.D. = F.V. – C.V.

= x – 5496

STEP 4 :

B.D. = Interest on F.V. for 158 days @ 14% p.a

$$x - 5496 = \text{'x'} \times \frac{219}{365} \times \frac{14}{100}$$

$$x - 43500 = \frac{21x}{250}$$

$$x - \frac{21x}{250} = 5496$$

$$\frac{229x}{250} = 5496$$

x = ₹ 6000

02.

$n = 25$, $\sum x = 125$, $\sum x^2 = 650$; $\sum y = 100$, $\sum y^2 = 460$, $\sum xy = 508$

It was however discovered two pairs (6,14) and (8,6) were incorrect while correct pairs were (8,12) and (6,8) . Obtain correct value of correlation coefficient

SOLUTION

INCORRECT	CORRECT
(6,14)	(8,12)
(8,6)	(6,8)

STEP 1 :

$$\Sigma x = 125 - (6 + 8) + (8 + 6) = 125$$

$$\Sigma y = 100 - (14 + 6) + (12 + 8) = 100$$

$$\Sigma x^2 = 650 - (6^2 + 8^2) + (8^2 + 6^2) = 650$$

$$\Sigma y^2 = 460 - (14^2 + 6^2) + (12^2 + 8^2)$$

$$= 460 - 232 + 208 = 436$$

$$\Sigma xy = 508 - (84 + 48) + (96 + 48) = 520$$

STEP 2 :

$$r = \frac{n\Sigma xy - \Sigma x\Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{25(520) - (125)(100)}{\sqrt{25(650) - (125)^2}\sqrt{25(436) - (100)^2}}$$

$$= \frac{13000 - 12500}{\sqrt{16250 - 15625}\sqrt{10900 - 10000}}$$

$$= \frac{500}{\sqrt{625}\sqrt{900}}$$

$$= \frac{500}{25 \times 30}$$

$$= \frac{20}{30}$$

$$= 0.67$$

03.

x	lx	dx	qx	px	Lx	Tx	ex⁰
20	88230	?	?	?	?	?	?
21	79473	-	-	-	-	3205552	?

Notations have the usual meaning . Complete the table .

SOLUTION

$$d_x = l_x - l_{x+1}$$

$d_{20} = l_{20} - l_{21}$

$= 88230 - 79473$

$= 8757$

$$q_x = \frac{d_x}{l_x}$$

$q_{20} = \frac{d_{20}}{l_{20}} = \frac{8757}{88230}$

$= 0.09924$

LOG CALC
3.9423
− 4.9456
AL $\overline{2}$.9967
0.09924

$$p_x = 1 - q_x$$

$p_{20} = 1 - q_{20}$

$= 1 - 0.09924$

$= 0.90076$

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$L_{20} = \frac{l_{20} + l_{21}}{2}$

$= \frac{88230 + 79473}{2}$

$= 83851.5$

$= 83852$

$$T_{x+1} = T_x - L_x$$

$T_{21} = T_{20} - L_{20}$

$3205552 = T_{20} - 83852$

$T_{20} = 3205552 + 83852$

$T_{20} = 3289404$

$$e_x^0 = \frac{T_x}{l_x}$$

$e_{20}^0 = \frac{T_{20}}{l_{70}} = \frac{3289404}{88230}$

$= 37.28$

LOG CALC
6.5171
− 4.9456
AL 1.5715
37.28

$e_{21}^0 = \frac{T_{21}}{l_{21}} = \frac{3205552}{79473}$

$= 40.33$

LOG CALC
6.5059
− 4.9002
AL 1.6057
40.33

ans :

x	lx	dx	qx	px	Lx	Tx	ex⁰
20	88230	**8757**	**0.09924**	**0.90076**	**83852**	**3289404**	**37.28**
21	79473	---	----	----	---	3205552	**40.33**

Q6. (A) Attempt any TWO of the following **(06)**

01. for a group of 30 couples the regression line of age of wife (y) on the age of husband (x) is given as $3y - 4x + 60 = 0$. Ratio of variance of x to variance of y is 9:25 and the mean age of wife is 40 years , find the correlation coefficient and the mean age of husbands

SOLUTION

STEP 1 : y on x :

$$3y - 4x + 60 = 0$$

$$3y = 4x - 60$$

$$y = \frac{4}{3}x - \frac{60}{3}$$

$$b_{yx} = \frac{4}{3}$$

STEP 2 : $b_{yx} = r\dfrac{\sigma y}{\sigma x}$

$$\frac{4}{3} = r\,\frac{5}{3} \qquad \text{GIVEN : } \frac{\sigma x^2}{\sigma y^2} = \frac{9}{25}$$

$$r = \frac{4}{5}$$

STEP 3 : sub y = 40 in

$$3y - 4x + 60 = 0$$

$$120 - 4x + 60 = 0$$

$$4x = 180$$

$$x = 45$$

mean age of husbands = 45 yrs

02. Find solution set of the in equation $\dfrac{x+4}{2x-1} \geq 3$.
Represent it on the number line

$$\frac{x+4}{2x-1} \geq 3$$

$$\frac{x+4}{2x-1} - 3 \geq 0$$

$$\frac{x + 4 - 6x + 3}{2x-1} \geq 0$$

$$\frac{7-5x}{2x-1} \geq 0$$

CASE 1 $7 - 5x \geq 0$ & $2x - 1 > 0$

$7 \geq 5x$ & $2x > 1$

$x \leq 1.4$ & $x > 0.5$

$0.5 < x \leq 1.4$

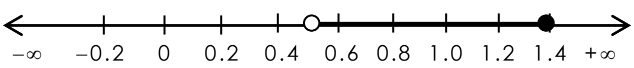
$x \in (0.5 , 1.4]$

CASE 2 $7 - 5x \leq 0$ & $2x - 1 < 0$

$7 \leq 5x$ & $2x < 1$

$x \geq 1.4$ & $x < 0.5$

NOT POSSIBLE , Pl. DISCARD



03. a team of 4 horses and 4 riders has entered the jumping show contest . The number of penalty points to be expected when each rider rides horse is shown below . How should the horses be assigned to the riders so as to minimize the expected loss . Also find the minimum expected loss

HORSES

RIDERS		H_1	H_2	H_3	H_4
	R_1	12	3	3	2
	R_2	1	11	4	13
	R_3	11	10	6	11
	R_3	5	8	1	7

SOLUTION

$$\begin{array}{cccc} 10 & 1 & 1 & 0 \\ 0 & 10 & 3 & 12 \\ 5 & 4 & 0 & 5 \\ 4 & 7 & 0 & 6 \end{array}$$

Reducing the matrix using 'ROW MINIMUM'

$$\begin{array}{cccc} 10 & 0 & 1 & 0 \\ 0 & 9 & 3 & 12 \\ 5 & 3 & 0 & 5 \\ 4 & 6 & 0 & 6 \end{array}$$

Reducing the matrix using 'COLUMN MINIMUM'

$$\begin{array}{cccc} 10 & \boxed{0} & 1 & \cancel{0} \\ \boxed{0} & 9 & 3 & 12 \\ 5 & 3 & \boxed{0} & 5 \\ 4 & 6 & \cancel{0} & 6 \end{array}$$

Allocation using 'SINGLE ZERO ROW COLUMN' method

allocation INCOMPLETE

REVISE THE MATRIX

$$\begin{array}{ccccc} & 10 & \boxed{0} & 1 & \cancel{0} \\ & \boxed{0} & 9 & 3 & 12 \\ \surd & 5 & 3 & \boxed{0} & 5 \\ \surd & 4 & 6 & \cancel{0} & 6 \\ & & & \surd & \end{array}$$

STEP 1 – Drawing minimum lines to cover ALL '0's

$$\begin{array}{cccc} 10 & 0 & 4 & 0 \\ 0 & 9 & 3 & 12 \\ 2 & 0 & 0 & 2 \\ 1 & 3 & 0 & 3 \end{array}$$

STEP 2 – REVISE THE MATRIX

reduce all the uncovered elements by its minimum & add the same at the intersection

$$\begin{array}{cccc} 10 & \cancel{0} & 4 & \boxed{0} \\ \boxed{0} & 9 & 3 & 12 \\ 2 & \boxed{0} & \cancel{0} & 2 \\ 1 & 3 & \boxed{0} & 3 \end{array}$$

Re – allocation

Since every row and every column contains an ASSIGNED ZERO ,
The ASSIGNMNET PROBLEM is SOLVED

OPTIMAL ASSIGNMENT : $R_1 - H_4$; $R_2 - H_1$; $R_3 - H_2$; $R_4 - H_3$

Minimum Penalty points = 2 + 1 + 10 + 1 = 14

(B) Attempt any TWO of the following **(08)**

01. Two samples from bivariate populations have 15 observations each . The sample means of X and Y are 25 and 18 respectively . The corresponding sum of squares of deviations from means are 136 and 148 . The sum of product of deviations from respective means is 122 . Obtain the equation of line of regression of X on Y

$\overline{x} = 25$, $\overline{y} = 18$, $\Sigma(x - \overline{x})^2 = 136$, $\Sigma(y - \overline{y})^2 = 148$

$\Sigma(x - \overline{x})(y - \overline{y}) = 122$

$$b_{xy} = \frac{\Sigma(x - \overline{x})(y - \overline{y})}{\Sigma(y - \overline{y})^2}$$

$$= \frac{122}{148}$$

$$= 0.8243$$

LOG CALC

2.0864
− 2.1703
AL $\overline{1}.9161$
0.8243

$$x - \overline{x} = b_{xy}(y - \overline{y})$$

$$x - 25 = 0.8243(y - 18)$$

$$x - 25 = 0.8243y - 14.8374$$

$$x = 0.8243y - 14.8374 + 25$$

$$x = 0.8243y + 10.1626$$

02. Find mean and variance of the continuous random variable X whose p.d.f. is given by

$$f(x) = 6x(1 - x) \quad ; \quad 0 < x < 1$$
$$= 0 \quad ; \quad \text{otherwise}$$

i) $$E(X) = \int_0^1 x.f(x)\, dx$$

$$= \int_0^1 x.6x(1 - x)\, dx$$

$$= 6\int_0^1 x^2(1 - x)\, dx$$

$$= 6\int_0^1 (x^2 - x^3)\, dx$$

$$= 6\left(\frac{x^3}{3} - \frac{x^4}{4}\right)_0^1$$

$$= 6\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= 6\left(\frac{1}{12}\right)$$

$$= \frac{1}{2} \qquad = 0.5$$

ii) $$Var(X) = \int_0^1 x^2.f(x)\, dx - \left[E(X)\right]^2$$

$$= \int_0^1 x^2.6x(1 - x)\, dx - \frac{1}{4}$$

$$= 6\int_0^1 (x^3 - x^4)\, dx - \frac{1}{4}$$

$$= 6\left(\frac{x^4}{4} - \frac{x^5}{5}\right)_0^1 - \frac{1}{4}$$

$$= 6\left(\frac{1}{4} - \frac{1}{5}\right) - \frac{1}{4}$$

$$= 6\left(\frac{1}{20}\right) - \frac{1}{4}$$

$$= \frac{3}{10} - \frac{1}{4}$$

$$= \frac{1}{20}$$

$$= 0.05$$

03. Find the sequence that minimizes total elapsed time (in hours) required to complete the following jobs on three machines M_1 , M_2 and M_3 in the order $M_1M_2M_3$. Also find the minimum elapsed time and idle time for all three machines

Job	A	B	C	D	E
M_1	5	7	6	9	5
M_2	2	1	4	5	3
M_3	3	7	5	6	7

STEP 1 : Min time on M_1 = 5 ;

Max time on M_2 = 5

Min time on M_3 = 3

Min (M_1) $\geq$ Max (M_2) ……… condition satisfied to convert 3 m/c's to 2 m/c's

STEP 2 : CONVERTING TO 2 FICTITIOUS M/C'S G & H

G = M_1 + M_2

H = M_2 + M_3

Job	A	B	C	D	E
G	7	8	10	14	8
H	5	8	9	11	10

STEP 3 : OPTIMAL SEQUENCE

Min time **=** 5 on job A on machine H . Place the job at the end of the sequence

				A

Next min time = 8 on job B & E on machine G . Place them randomly at the start of the sequence

B	**E**			**A**

Next min time = 9 on job C on machine H . Place it at the end of the sequence before A

B	**E**		**C**	**A**

OPTIMAL SEQUENCE

B	**E**	**D**	**C**	**A**

STEP 4 : WORK TABLE

Job	B	E	D	C	A		total process time
M_1	7	5	9	6	5	=	32 hrs
M_2	1	3	5	4	2	=	15 hrs
M_3	7	7	6	5	3	=	28 hrs

JOBS	M_1		IDLE	M_2		IDLE	M_3		IDLE
	IN	OUT	TIME	IN	OUT	TIME	IN	OUT	TIME
						7			8
B	0	7	--	7	8	4	8	15	--
E	7	12	--	12	15	6	15	22	4
D	12	21	--	21	26	1	26	32	--
C	21	27	--	27	31	1	32	37	--
A	27	32	8	32	34	6	37	40	--

STEP 5 : Total elapsed time T = 40 hrs

Idle time on M_1 = T – (sum of processing time of all 5 jobs on M1)

= 40 – 32

= 8 hrs

Idle time on M_2 = T – (sum of processing time of all 5 jobs on M2)

= 40 – 15

= 25 hrs (CHECK – 7 + 4 + 6 + 1 + 1 + 6 = 25)

Idle time on M_3 = T – (sum of processing time of all 5 jobs on M3)

= 40 – 28

= 12 hrs (CHECK – 8 + 4 = 12)

DO NOT STOP

GET READY FOR NEXT